

Innovative Channel Estimation Method for Rayleigh, Rician, and Nakagami Channels: The Modified M-Estimator-I

Prakhar Hari¹, Sanjay Mathur², Prachi Mishra³, Pooja Singh⁴, Hari Kumar Singh^{5*}

¹Department of Electronics and Communication Engineering, G B P U A T Pantnagar

²Department of Electronics and Communication Engineering, G B P U A T Pantnagar

³Department of Electronics and Communication Engineering, G B P U A T Pantnagar

⁴Department of Electrical Engineering, Delhi Technological University, Delhi

^{5*}Department of Electronics and Communication Engineering, M J P Rohilkhand University Bareilly

Email: ¹prakharhari6766@gmail.com, ²mathur_sanjay@rediffmail.com, ³mishraprachiece@gmail.com, ⁴er.poojadc11@gmail.com

Corresponding Author: ^{5*}hari.singh@mjpru.ac.in

Abstract: In modern wireless communication, achieving higher data rates, reliability, and spectral efficiency necessitates advanced transmission technologies. These technologies face challenges from unpredictable wireless propagation environments, particularly channel fading, which causes signal strength variations. Accurate channel estimation techniques are crucial for adapting transmission parameters to current channel conditions and optimizing performance. Orthogonal Frequency Division Multiplexing (OFDM) mitigates multipath propagation effects, and Multiple-Input Multiple-Output (MIMO) technology enhances capacity and reliability. The combined MIMO-OFDM framework relies on precise channel estimation, but traditional methods often fail in dynamic, complex channels. This research introduces a novel channel estimation technique, termed Modified M-estimator-I, which provides enhanced accuracy and resilience in estimating channel state information. Building on robust statistical estimation principles, the Modified M-estimator-I adapts to channel variations, offering more reliable performance across Rayleigh, Rician, and Nakagami fading environments. This innovative approach addresses the limitations of conventional estimators by incorporating advanced statistical methodologies to mitigate outliers and noise, thus optimizing the estimation process in diverse fading conditions.

Index: - Channel Estimation, M-estimator-I, Fading Channel, Pilot-symbol based Channel estimation

1. Introduction

In the realm of modern wireless communication, the quest for higher data rates, increased reliability, and enhanced spectral efficiency has driven significant advancements in transmission technologies. However, these advancements are continually challenged by the unpredictable and often adverse nature of wireless propagation environments. Channel fading, a phenomenon characterized by variations in signal strength over time and space, poses a formidable obstacle to the reliable transmission of data. To counteract these effects, accurate channel estimation techniques are indispensable, as they enable the adaptation of transmission parameters to the prevailing channel conditions, thereby optimizing performance.

Orthogonal Frequency Division Multiplexing (OFDM) has emerged as a cornerstone technology in contemporary wireless communication systems, renowned for its robustness against inter-symbol interference (ISI) and inter-carrier interference (ICI). By transmitting data across multiple orthogonal subcarriers, OFDM effectively mitigates the impact of multipath propagation. Transmitted symbols can be accurately recovered through effective channel estimation, which corrects the channel impulse response (CIR) and determines the channel's impact on the transmitted symbols. [1] MLE and MMSEE techniques for CIR estimation in OFDM systems was discussed. [2] The challenge of large CSI feedback in massive MIMO systems by proposing an efficient scheme using spatial correlation and compressed sensing (CS) techniques was discussed in [3]. Hence better reconstruction of transmitted symbols is required.

Complementing OFDM, Multiple-Input Multiple-Output (MIMO) technology leverages multiple transmitting and receiving antennas to enhance system capacity and reliability through spatial diversity and spatial multiplexing. The synergy between OFDM and MIMO, known as MIMO-OFDM, represents a powerful framework for high-performance wireless communication. Despite these technological advancements, the effectiveness of MIMO-OFDM systems is fundamentally dependent on precise channel estimation. Traditional channel estimation techniques, including pilot-based and blind estimation methods, have been widely studied and implemented. Various channel estimation techniques were studied for wireless communication system [4]. Least square channel estimation in MIMO-OFDM systems using block-type and comb-type pilot structures investigated in [5]. The estimation of channel parameters in OFDM and generalized frequency division multiplexing has been investigated in [6] and [7]. A combined approach of channel estimation and multi-user detection (MUD) in MIMO-OFDM systems to mitigate multi-access interference and reduce BER is proposed in [8]. LS and LMMSE-based channel techniques for channel parameters were compared in [9]. These conventional methods often fall short in addressing the dynamic and complex nature of wireless channels, particularly in the presence of severe fading. This necessitates the development of more sophisticated and adaptive estimation techniques. Robust multiuser detection using M-estimation in wireless communication systems was analysed in [10]. M-estimator-based approach compared with LS and MMSE approach in Rayleigh and Rician fading channel is done in [11]. Different channel estimation techniques are generally used for massive MIMO system, which are studied in [12].

In [13], a channel estimation technique for OFDM systems with fading channels is proposed. Rayleigh fading is commonly observed in environments with a high density of scatters, such as urban areas, where a direct line of sight between the transmitter and the receiver is absent. This results in a signal that is composed of numerous reflected and scattered paths, each with a different phase and amplitude. [14-15] proposes a method for analysing the performance of OFDM systems over Rayleigh fading channels, considering the effects of phase noise and carrier frequency offset. Rician fading, on the other hand, occurs in scenarios where there is a dominant line of sight path in addition to the scattered paths, leading to a more stable signal compared to Rayleigh fading. Nakagami fading provides a versatile model that can describe a wide range of fading conditions, from severe fading like Rayleigh to mild fading closer to Rician.

This research introduces a novel channel estimation technique, termed Modified M-estimator-I. The Modified M-estimator-I builds upon the principles of robust statistical estimation to provide enhanced accuracy and resilience in estimating channel state information. Unlike conventional estimators, which may be sensitive to outliers and noise, the Modified M-estimator-I incorporates advanced statistical methodologies to adapt to the variations in the channel conditions, offering a more reliable and robust performance across diverse fading environments.

In this paper, Section II outlines the system model. Section III explains the various channel estimation techniques and introduces the proposed Modified M-estimator-I. Section IV presents the simulation results, and Section V offers concluding remarks. The paper concludes with a list of references.

2. System MODEL

(A) OFDM and MIMO-OFDM System

Orthogonal Frequency Division Multiplexing (OFDM) is a wireless technology that transmits data on orthogonal subcarriers, allowing symbols to be closely packed without interference, thereby saving bandwidth. It uses Multi-Carrier Modulation to partition high-rate data into parallel bit streams, modulated by orthogonal subcarriers with IFFT and FFT for modulation and demodulation. Guard bands, cyclic prefixes, and pilot symbols are used to eliminate ISI, ICI, and assist in channel estimation. OFDM is robust against ISI, ICI, and fading, with enhanced performance when coupled with Multiple-Input Multiple-Output (MIMO) systems. MIMO employs multiple transmitting and receiving antennas to enhance system capacity and coverage through spatial multiplexing and diversity techniques. MIMO-OFDM improves link reliability and spectrum efficiency by increasing signal gain, reducing multi-path fading, and boosting channel capacity and SNR through spatial multiplexing and diversity coding.

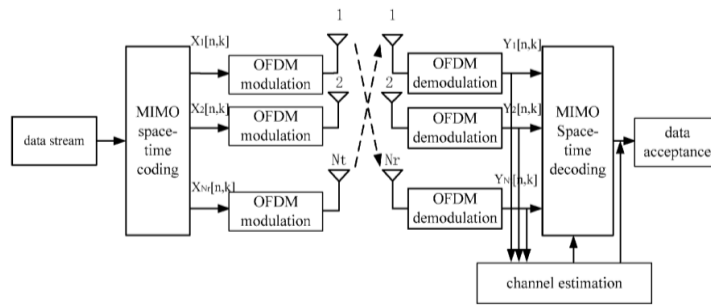


Fig. 1 Block diagram of MIMO-OFDM System

(B) Channel Estimation in Single Input Single Output (SISO) System

This analysis focuses on a single input, single output (SISO) system. Channels can affect different frequency signals in unique ways, making it essential to estimate the channel for each frequency channel. Due to the potentially large number of channels, this task can become complex and resource-intensive. Therefore, channel estimation is often performed on a subset of channels, with the estimates for the remaining channels being interpolated from these calculated values.

Consider selecting three frequencies, f_1 , f_2 and f_3 from all the available frequencies. The corresponding reference or pilot signals will be $x(f_1)$, $x(f_2)$, and $x(f_3)$. These reference signals are complex, consisting of I/Q data. For the other frequency channels, we can interpolate the channel characteristics using the calculated ones. When these reference signals are received at the destination, they will include distortions and noise, represented by $y(f_1)$, $y(f_2)$, and $y(f_3)$. To express the received signals $y(f)$ in terms of $x(f)$, a channel function $h(f)$ is required for the specific frequency. The relationship between the reference signal, the received signal, and the channel function can thus be represented by the correlation function.

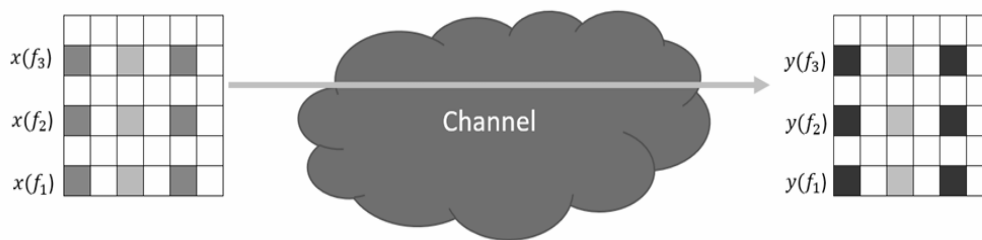


Fig. 2 Single Input Single Output System

Which can be written as

$$y(f_1) = h(f_1) \cdot x(f_1)$$

$$y(f_2) = h(f_2) \cdot x(f_2)$$

$$y(f_3) = h(f_3) \cdot x(f_3)$$

....(1)

from these equations, $x(f)$ and $y(f)$ are known thus $h(f)$ can be calculated

$$h(f_1) = y(f_1) \cdot x^H(f_1)$$

$$h(f_2) = y(f_2) \cdot x^H(f_2)$$

$$h(f_3) = y(f_3) \cdot x^H(f_3)$$

....(2)

Here $x^H(f)$ is the Hermitian of $x(f)$.

Since only three frequencies have been considered and channel characteristics have been estimated for these frequencies, the properties for the remaining frequencies can be interpolated from the known characteristics. The channel function in these equations represents channel distortion. Additionally, noise is added to the distorted signals, so the actual equations of the received signal are as follows:

$$y(f_1) = h(f_1) \cdot x(f_1) + n(f_1)$$

$$y(f_2) = h(f_2) \cdot x(f_2) + n(f_2)$$

$$y(f_3) = h(f_3) \cdot x(f_3) + n(f_3)$$

....(3)

Like how channel function was estimated, theoretically noise can also be estimated by using averaged channel estimate $\hat{h}(f)$.

$$\begin{aligned} n(f_1) &= y(f_1) - \hat{h}(f_1) \cdot x(f_1) \\ n(f_2) &= y(f_2) - \hat{h}(f_2) \cdot x(f_2) \\ n(f_3) &= y(f_3) - \hat{h}(f_3) \cdot x(f_3) \end{aligned} \dots(4)$$

However, absolute values of noise are not helpful for channel estimation due to the continuous variations in noise within a channel. What is beneficial is an estimated function of noise that accounts for and models these variations. Various algorithms and methods can achieve this. One such method, implemented in srsLTE (an open-source LTE implementation on SDR), involves subtracting the averaged channel estimate from the actual channel estimate.

$$\begin{aligned} n(f_1) &= h(f_1) - \hat{h}(f_1) \\ n(f_2) &= h(f_2) - \hat{h}(f_2) \\ n(f_3) &= h(f_3) - \hat{h}(f_3) \end{aligned} \dots(5)$$

(C) Channel Estimation in Multiple Input Multiple Output (MIMO) System

In a multiple input multiple output (MIMO) system, there are multiple transmission endpoints and multiple reception endpoints.

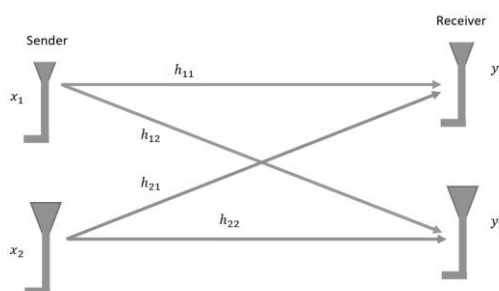


Fig. 3 A general 2x2 MIMO System

In a MIMO system, the channel estimation process is similar, but now there are two signals received from a single source. This indicates that two distinct paths in the medium were used, one for each signal. Therefore, to compute the final signal $y(f)$ for each frequency, both received signals must be considered. This results in the formation of a matrix of received signals.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \dots(6)$$

Similar to a SISO system, in a MIMO system, the Hermitian of the input matrix x can be taken to estimate the channel matrix h . Similarly, the noise matrix n can also be calculated using matrix operations.

3. Channel Estimation

(A) Least Square (LS) Channel Estimation

Least squares (LS) channel estimation is a fundamental technique used in wireless communication systems to estimate channel state information (CSI). It is one of the simplest and most widely used methods due to its straightforward implementation and solid theoretical foundation. The LS channel estimation method aims to minimize the sum of the squared differences between the observed received signals and the predicted signals based on the estimated channel. This approach leads to a straightforward and intuitive solution, making it easy to implement.

LS channel estimation for 5G multicarrier wireless communications in Rayleigh, Rician and Nakagami fading channel is given by

$$\hat{H}_{LS} = (X^H X)^{-1} X^H Y = X^{-1} Y \dots(7)$$

Where estimate of H is \hat{H} and X^H is the Hermitian matrix of X .

(B) Minimum Mean Square Error (MMSE) Channel Estimation

Minimum Mean Square Error (MMSE) channel estimation is an advanced technique used in wireless communication systems to estimate channel state information (CSI) with improved accuracy and robustness compared to simpler methods like Least Squares (LS) estimation. MMSE channel estimation aims to minimize the mean square error between the actual and estimated channel responses by incorporating statistical knowledge

of the channel and noise. This method leverages both the received signal and prior information about the channel statistics to achieve a more accurate estimate.

MMSE channel estimation, the received signal y is modelled as

$$y = Hx + n,$$

where H is the channel matrix to be estimated, x is the transmitted signal, and n is the noise. The MMSE estimate \hat{H} is given by:

$$\hat{H}_{MMSE} = R_H X^T (X R_H X^T + \sigma^2 I)^{-1} y$$

....(8)

Where R_H is the autocorrelation matrix of the channel, σ^2 is the noise variance, and I is the identity matrix.

(C) M-Estimator Channel Estimation

An M-estimator [16] is employed to enhance the robustness of channel estimation techniques in 5G wireless communication systems. The penalty, influence, and weight functions of the M-estimator for channel estimation in 5G multicarrier wireless communication over Rayleigh and Rician fading channels are provided by

$$\rho_{M-estimator}(x) = \begin{cases} \frac{x^2}{2} & \text{for } |x| \leq a \\ a^2 - a|x| & \text{for } a < |x| \leq b \\ \frac{-ab}{2} x \exp\left(1 - \frac{x^2}{b^2}\right) + d & \text{for } |x| > b \end{cases}$$

....(9)

$$\psi_{M-estimator}(x) = \begin{cases} x & \text{for } |x| \leq a \\ a \operatorname{sign}(x) & \text{for } a < |x| \leq b \\ \frac{a}{b} x \exp\left(1 - \frac{x^2}{b^2}\right) & \text{for } |x| > b \end{cases}$$

....(10)

$$w_{M-estimator}(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ \frac{a \operatorname{sgn}(x)}{x} & \text{for } a < |x| \leq b \\ \frac{a}{b} \exp\left(1 - \frac{x^2}{b^2}\right) & \text{for } |x| > b \end{cases}$$

....(11)

Where d is a constant. The choice of the constants $a(=kv^2)$ and $b(=2kv^2)$ depends on the robustness measures derived from the influence function (k is also any constant). A robust estimator should possess a finite value of c . M- estimators reduce the outcome of outliers with $\min \sum_i \rho(x_i)$, where $\rho(x)$ is penalty function, influence function $\psi(x)$ is the derivative of penalty function and $w_{Mestimator}(x) = \frac{\psi(x)}{x}$ is the weight function. The penalty function, influence function and weight function of M-estimator are shown in figures 4 – 6

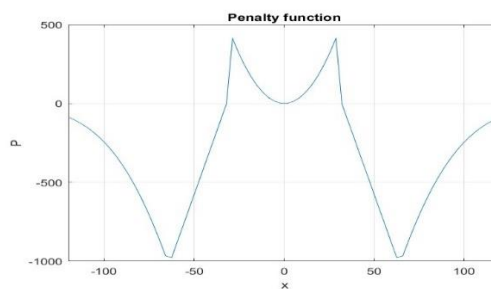


Fig. 4 Penalty Function of M-estimator

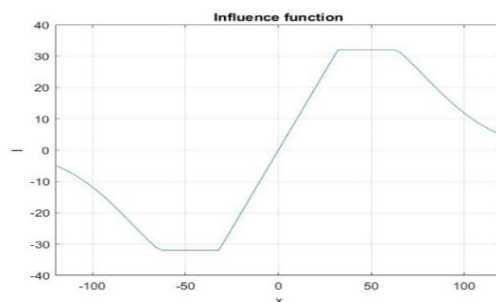


Fig. 5 Influence Function of M-estimator

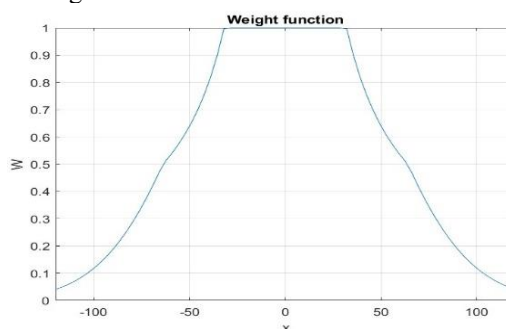


Fig. 6 Weight Function of M-estimator

M-estimator channel estimation for 5G multicarrier wireless communications in Rayleigh, Rician and Nakagami fading channel is given by

$$\hat{H}_{M-Estimator} = (X^H \psi(x) X)^{-1} X^H Y \dots(12)$$

Where $\psi(x)$ is influence function of the estimator described in above section. X is the transmitted signal in the channel. H is Hermitian transform matrix. Y represents the received signal in the channel.

(D) Modified M-estimator-I

From the statistics we know that the M-estimator is a statistical estimator used in robust statistics to estimate parameters of a statistical model. It is particularly useful when the data may contain outliers or other departures from the assumed model. In the context of wireless channel estimation, the M-estimator approach can be applied to estimate parameters such as the channel impulse response (CIR), the channel frequency response (CFR), or the channel matrix in MIMO systems. The previously proposed M-estimator based channel estimation algorithm uses the influence function of M-estimator. The modification proposed in M-estimator, which is named as Modified M-estimator-I uses ratio function instead of influence function.

The ratio function is described as the ratio of influence function to penalty function or object function. The ratio function is obtained as $R_{MODIFIED}(x) = \frac{\psi_{PROPOSED}(x)}{\rho_{PROPOSED}(x)}$. The proposed ratio function equation is described below

$$R_{MODIFIED}(x) = \begin{cases} \frac{2}{x} & \text{for } |x| \leq a \\ \frac{sign(x)}{a - |x|} & \text{for } a < |x| \leq b \\ \frac{\frac{a}{b}x \exp\left(1 - \frac{x^2}{b^2}\right)}{\frac{-ab}{2}x \exp\left(1 - \frac{x^2}{b^2}\right) + d} & \text{for } |x| > b \end{cases}$$

....(13)

The Modified M-estimator 2 based method finds the channels estimation parameters using the below equation

$$\hat{H}_{MODIFIED-I} = (X^H R(x) X)^{-1} X^H Y \dots(14)$$

Where The R(x) is the ratio function described in above section. Ratio function is piecewise function, where the expression changes depending on the interval in which x lies. The X is the transmitted signal in the channel. H is Hermitian transpose matrix. Y represents the received signal in the channel. Figure... shows the ratio function of proposed Modified M-estimator-I.

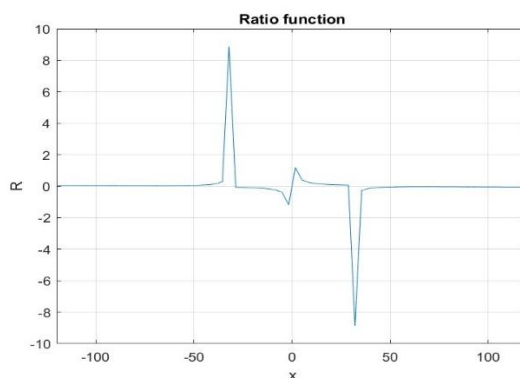


Fig. 7 Ratio Function of Proposed Modified M-estimator-I

4. Simulation Results

In simulations, channel estimation techniques based on LS (Least Squares), MMSE (Minimum Mean Square Error), M-estimator, and Modified M-estimator-I are compared. The performance evaluation of these techniques is conducted in terms of Mean Squared Error (MSE) and Signal-to-Noise Ratio (SNR). Figures 8 to 10 present the MSE and SNR results of these estimation techniques in Rayleigh fading, Rician fading, and Nakagami fading channels, respectively. It is evident across all these algorithms that MSE decreases as SNR increases, indicating better performance at higher SNR levels. Simulation results demonstrate that the Modified M-estimator-I outperforms LS, MMSE, and M-estimator techniques in channel estimation. All coding and simulations for channel estimation are implemented in MATLAB. The simulation parameters are detailed in the table below.

Table-1 Details of Simulation Parameters

| Sr. No. | Parameters | Value(s) |
|---------|--|---|
| 1 | Number of base station antennas | 2 |
| 2 | Number of UE antennas | 2 |
| 3 | Number of the resource block | 6 |
| 4 | Number of the resource bock per signal carrier | 12 |
| 5 | Number of transmitted symbols | 2000 |
| 6 | DFT size | Number of resource block X number of resource blocks per single carrier |
| 7 | FFT size | 500 |
| 8 | OFDM CP | Normal |
| 9 | Power delay profile | [1, 0.5, 0.25, 0.125] |
| 10 | Channel | Rayleigh fading channel, Rician fading channel, Nakagami fading channel |
| 11 | Channel estimation algorithms | LS, MMSE, M-estimator, Modified M-estimator |

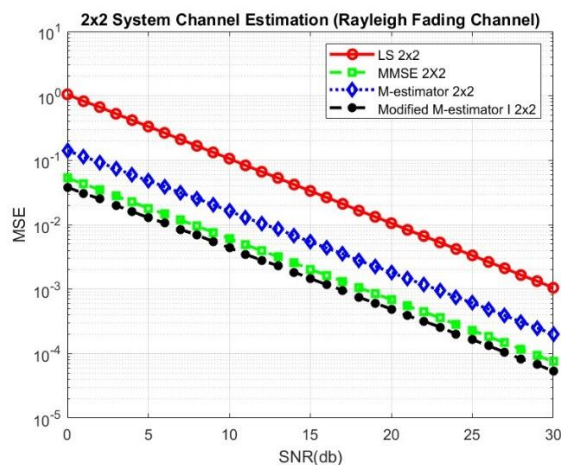


Fig. 8 BER vs SNR Performance of LS, MMSE, M-estimator and Proposed modified M-estimator-I in Rayleigh Fading channel.

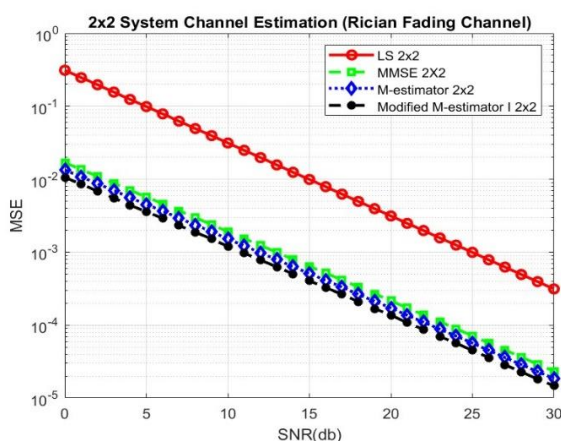


Fig. 9 BER vs SNR Performance of LS, MMSE, M-estimator and Proposed modified M-estimator-I in Rician Fading channel.

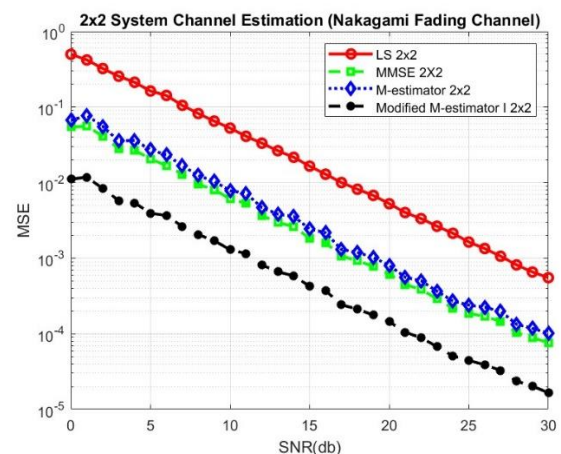


Fig. 10 BER vs SNR Performance of LS, MMSE, M-estimator and Proposed modified M-estimator I in Nakagami Fading channel.

5. Conclusion

In this paper, analysis of a Modified M-estimator-I based channel estimation technique for OFDM wireless communication systems in Rayleigh, Rician, and Nakagami fading channels is done. The newly proposed Modified M-estimator-I is employed to optimize the channel estimation process. To validate its effectiveness, we

provide simulation results that compare the performance of our proposed technique against the LS, MMSE, and M-estimator techniques. Based on these simulations, it is observed that the proposed Modified M-estimator-I based channel estimation technique demonstrates superior performance compared to LS, MMSE, and M-estimator techniques across Rayleigh, Rician, and Nakagami fading channels.

6. References

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